How Accurate the NEST Price Is

ABSTRACT

This short article develops a model to estimate the difference between the NEST price and a source price, e.g. price from an exchange. Under plausible assumptions, we show that the difference can be as small as 0.003 when volatility is small. It can even be lower if the transaction cost in the blockchain gets lower.

1. Model Setup

A *price-provider* is an individual who inputs a price into the NEST system and waits for a certain number of blocks passing to be verified by other individuals. The operation is equivalent to write an American type call and put option that anyone else can exercise it by using the input price as the exercise price. Thus, the price-provider shall minimize the value of this option by carefully choosing an input price. Precisely, the price-provider's objective problem is

$$P^* = \arg\min_P \left(\max_\tau E^Q[e^{-r\tau}|S_\tau - P|] \right), \tag{1}$$

where $\tau \leq T_0$ is a stopping time and T_0 is a fixed time horizon[1], P is the input price decided by the price-provider. In other words, the price-provider has to minimize the value of one American type option by choosing an appropriate exercise price P. Here asset price S_t, $t \geq 0$ shall be referred to the price in an exchange at time t. Thus, the market is complete and we price the derivative in a risk-neutral framework by taking the expectation under the risk-neutral probability Q.

Denote the solution to the above problem by $P^* = P(S_0; \sigma)$, where σ is the volatility of the source price sequence S_t. Noting that the price-provider inputs a price optimally based on all of his information from a centralized market and/or from the decentralized world.

1.1 Arbitrageur

The price-provider writes an American option when he inputs a price K. It seems that anybody can exercise the option without any cost. However, the NEST requires that the one (arbitrageur) who exercises the derivative must input another price and lock in as much as β times the original asset requirement. In other words, to exercise one option, the arbitrageur

[1]For the NEST system, the time horizon actually is random because the time interval between two successive Ethereum blocks is. The framework in this note can be extended to study this case.

has to write β units of the same type of American options, where $\beta > 1$ is a specific multiplier.

One arbitrageur who wishes to make profit from the derivative can construct (sell) a portfolio in the outside market that replicates the derivative. Then the arbitrageur can make a risk-free profit the same as the value of the derivative. However, there is risk that the arbitrageur can not obtain the opportunity to exercise the derivative because it is competitive to take the arbitrage. Therefore, instead of making the risk-free profit, a realistic strategy is to make a *quick* profit in the sense of statistic arbitrage as follows.

The arbitrageur does nothing but waits until the difference between the outside asset price and the input price P is sufficiently large. Then he exercises the option and buys or sells in the exchange simultaneously to make money without any risk. Such an opportunity may not be available for all time, but in long time there are many chances. So statistically the arbitrageur can make money.

We calculate the following objective function for the arbitrageur:

$$\max_{\tau} E[(|S_\tau - P| - a)1_{|S_\tau - P| > A, \tau < T_0} \tag{2}$$

where A represents all costs of the transaction, including Ethereum transaction fee and the value of the derivative multiplied by β. The stopping time τ in the above indicates that the arbitrageur will wait for the best time to take the arbitrage. However, considering the competitive environment, most likely, the profit is taken when the first time a target is reached. So the objective function turns to be

$$E[(|S_\eta - K| - A)1_{\eta \leq T_0}] \,, \tag{3}$$

where $\eta = \inf\{t : |S_t - P| - A > \epsilon\}$ and ϵ is the minimum target profit of the arbitrageur. Along with the arbitrage-taking method (3), the corresponding loss (or the cost of inputing a price) of the price-provider is

$$E(|S_\eta - P| \, 1_{\eta \leq T_0}) \,.$$

The price-provider shall minimize the cost by choosing an appropriate K. That is, the objective function of the price-provider is

$$\min_P E[|S_\eta - P| \, 1_{\eta \le T_0}.$$

In fact, we should price it in a risk-neutral sense:

$$V^*(0) = \min_P E^Q[e^{-r\eta}|S_\eta - P| \, 1_{\eta \le T_0}] \, ,$$

where r is the risk-free interest rate. It yields that the price-provider can construct a portfolio in the outside market to hedge this derivative, so that his loss is a deterministic value same as V^*.

2. A Solution of the Model

Given the design of the NEST, we let

$$A = \beta V^*(\eta),$$

where $V^*(\eta)$ denotes value of the same derivative at time η. We let ϵ be the transaction fee in the blockchain (the gas fee).

Aware of the way the option is exercised, the price-provider actually considers the objective problem as follows.

$$V^*(0) = \min_P E^Q[e^{-r\eta}|S_\eta - P| \, 1_{\eta \le T_0}] = \min_P E^Q[e^{-r\eta}(A + \epsilon)1_{\eta \le T_0}] = \min_P E^Q[e^{-r\eta}(\beta V^*(\eta) + \epsilon)1_{\eta \le T_0}. \quad (4)$$

We assume that the asset price follows a Brownian motion with drift:

$$S_t = S_0 + \mu t + \sigma Z_t,$$

where Z_t is a standard Brownian motion. Then $V^*(\cdot)$ is identical at any time. The recursive formula (4) is simplified (for a stationary solution under constant state variables μ and σ)

$$V^* = \min_P E^Q[e^{-r\eta} 1_{\eta \leq T_0}] \, (\beta V^* + \epsilon). \tag{5}$$

Exploiting the density function of η, the first hitting time of Brownian motion, we can evaluate the expecation in (5) and solve for V^* and P^* numerically.

Set $\mu = r = 0$, $\epsilon = 0.003$ (the gas fee of one transaction in the Ethereum/10ETH), $S_0 = 1$, we obtain the following results.

For $\sigma = 0.0001,\ 0.001,\ 0.003$ per second:

$\beta = 1.5$: $V^* = 0.0030, 0.0104, 0.0327$; probability of arbitrage: 0.0726, 0.3353, 0.3765

$\beta = 2$: $V^* = 0.0003$, 0.0092, 0.0291; probability of arbitrage= 0.0792, 0.4301, 0.4755,

$\beta = 3$: $V^* = 0.0002$, 0.0074, 0.0233; probability of arbitrage= 0.0894, 0.6064, 0.6696, where the probability of arbitrage is defined by $E^Q[1_{\eta \leq T_0}]$. For all of these cases, the optimal input-price $P^* = S_0 = 1$. Since S_t is assumed to be a Brownian motion without a drift, this answer is obvious.

The sensitivity analysis regarding verification during time T_0, probability of arbitrage, β, volatility σ are shown in Figure 1 and 2.

2.1 Difference between NEST Price and Price of Exchange

By the preceding analysis, the difference between the NEST price and the price from an exchange is bounded by $a := \beta V^* + \epsilon$. Figure 3 indicates the upper bound can be as small as 0.003. The upper bound can be decreased if the transaction (arbitrage) cost in the blockchain becomes small. Alternatively, We may increase the asset requirement of inputing a price to decrease the relative weight of ϵ. For example, if we increase the asset requirement to 50 ETHs, the difference bound turns to be 0.002 only.

Figures

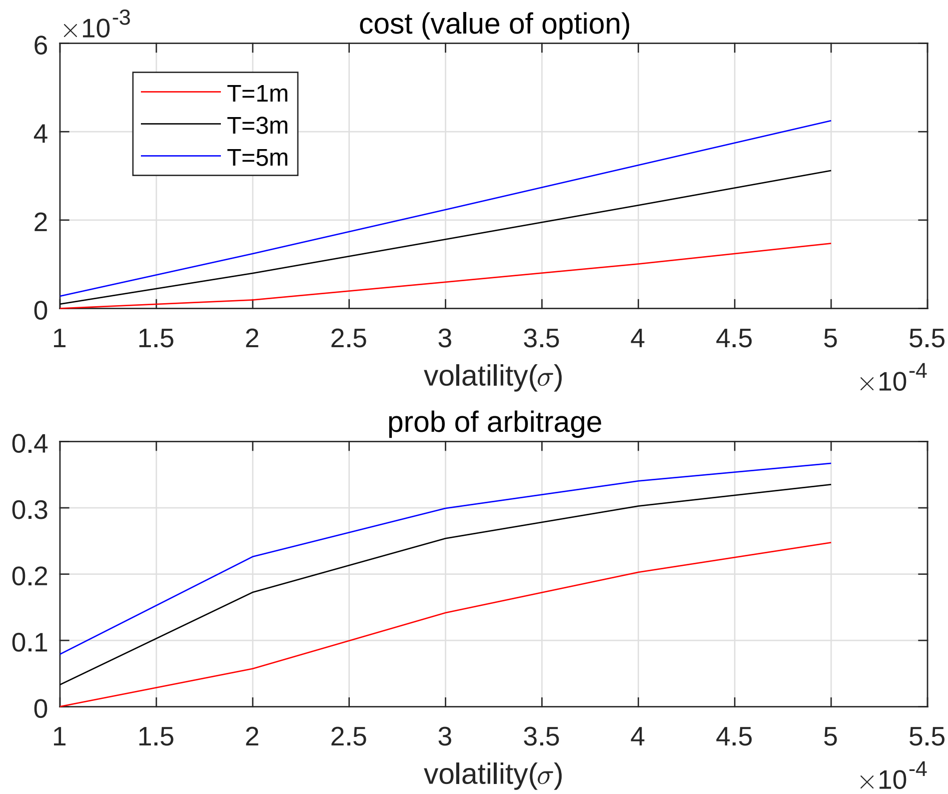

Figure 1. This figure depicts effects of volatility σ on cost of price-inputing and probability of arbitrage.

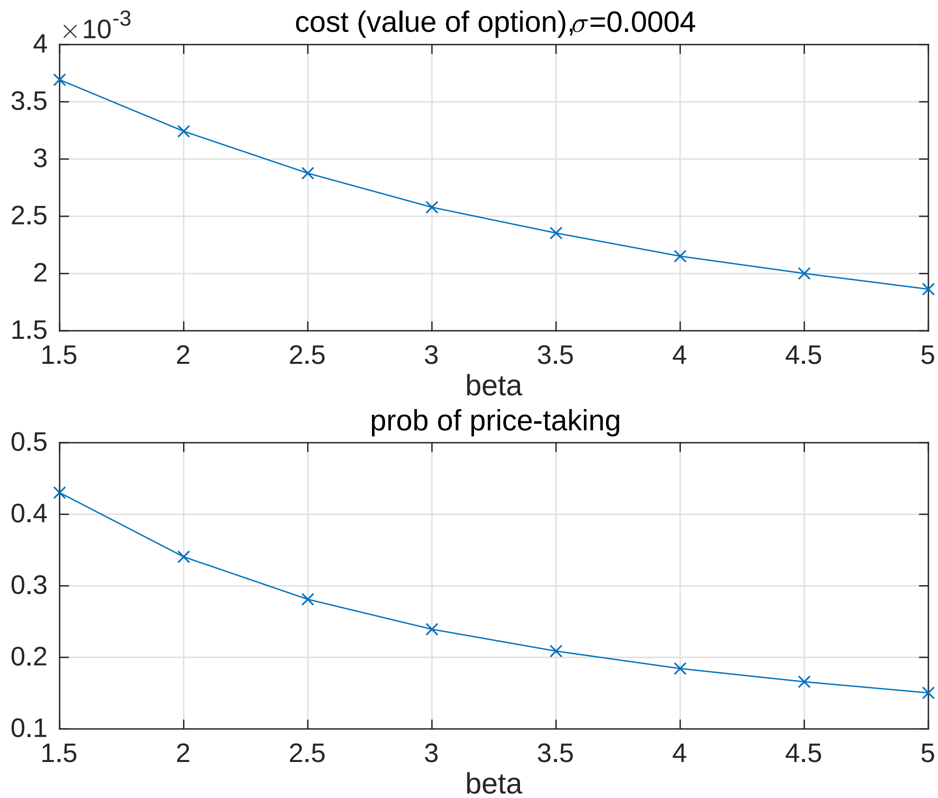

Figure 2. This figure depicts the effect of β on cost of price-inputing and probability of arbitrage.

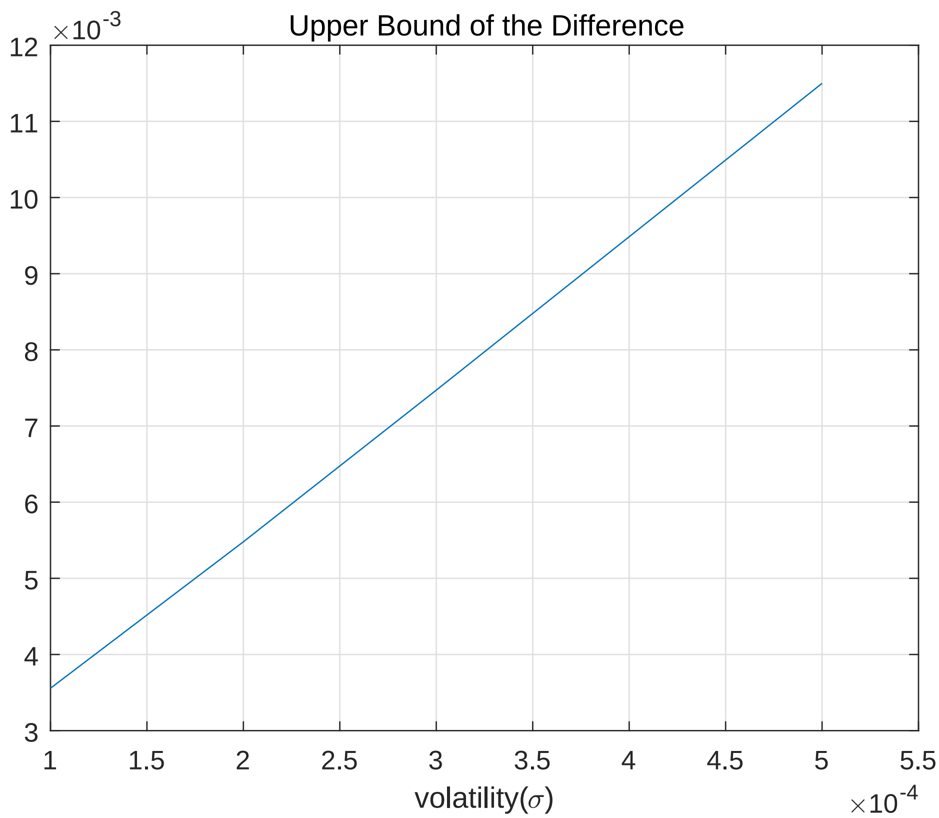

Figure 3. This figure shows the upper bound of difference between the NEST price and the price of an exchange at the same time.